Equilibrium Trade in Automobiles

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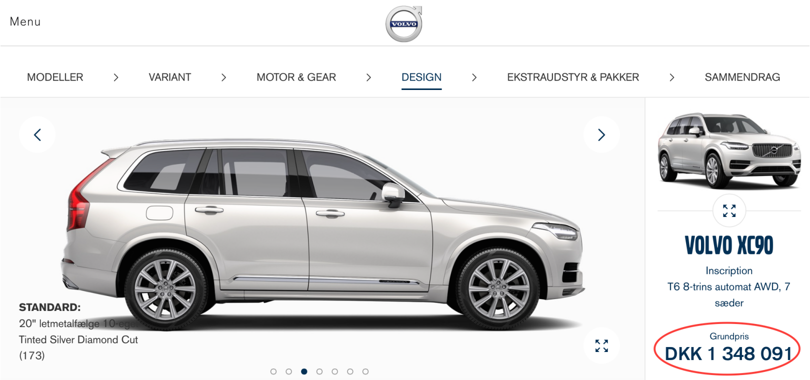
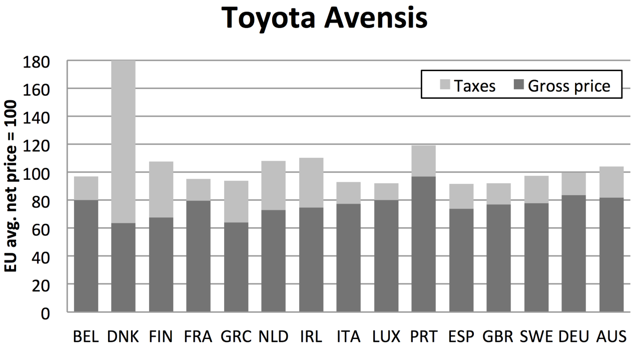
How much is a Volvo in Denmark?

About $200,000!



MSRP in US: $62,350

Danish car registration tax: 180%! (plus 25% VAT)



Policy question:
What is the best strategy to switch car taxes from purchase to usage?

Contributions of our analysis

- **Methodological contribution:** Compute *counterfactual dynamic stationary flow equilibria* in the auto market with primary/secondary market interactions, transaction costs and consumer heterogeneity
- **Computational framework:** we show how to rapidly compute dynamic equilibria with flexible specifications of transactions costs and consumer heterogeneity using a nested Newton algorithm
- **Econometric contribution:** *Doubly nested fixed point* (DNFXP) maximum likelihood estimator • 131 parameters • nested recalculation of dynamic equilibrium in likelihood • 4 car types and 8 consumer types • 39 millon observations of state transitions • Under 30 minutes on a laptop • Identification even though accidents and prices of used cars are unobserved
- **Our main finding:** Registration tax is "over the top" of the Laffer curve • Welfare improving tax policies that generate higher consumer welfare, government tax revenue, and reduce CO_2 emissions

Part I: Stationary equilibrium with transaction costs and consumer heterogeneity: theory

Part II: Modeling the Danish secondary market for automobiles

Model overview

- **Consumers:** Unit mass, infinitely lived, discrete types τ
  - *Ownership decisions:* keep, trade, purge + scrap or sell if applies
  - *Driving decicison:* how much to drive
- **Cars:** $j \in \{1, ..., J\}$ types of ages $a \in \{1, ..., \bar{a}\}$
- **Scrappage:** Forced (accidents & end of life) or by choice
  - Stochastic: due to accident with probability α_j
  - End of life: at scrappage at age $\bar{a}$
  - Endogenous: when getting rid of a car voluntarily
- **Idiosyncratic heterogeneity:** IID EV/GEV terms (non-degenerancy of choice probabilities $\Rightarrow$ existence of equilibrium)
- **Persistent heterogeneity:** Finite number of consumer types
- **Key simplification:** all dynamic effects through age of car and ownership states (driving has no dynamic implications)

DNFXP algorithm (roadmap)

- **Outer optimization:** Maximum likelihood search over θ
- **Inner equilibrium solver:** Find prices, $P^{\star}$, so $ED(P^{\star}, q(P^{\star})) = 0$
- **Excess demand:** Each trial value of P requires
  1. Solve single agent DP/fixed point given P
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Utility of car ownership and consumer heterogeneity

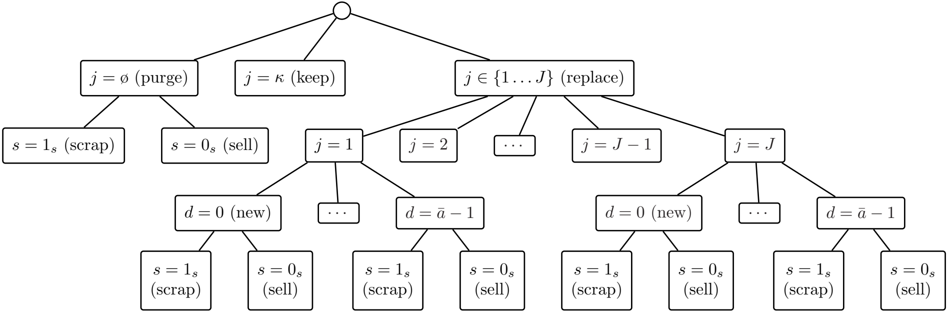
$$\text{Utility} = u(i,a) - \mu\big[\text{operating costs} + \text{trade and transaction costs}\big] + \epsilon$$

- Car utility $u(i,a)$ is a decreasing function of car age a that reflects
  - decreasing utility of car services
  - increasing cost of maintenance
- Marginal utility of money μ

Idiosyncratically heterogeneous consumers

- **Extreme value** consumer types (taste shifters)
- GEV specification for $\epsilon \rightarrow$ nested choices to allow correlation between alternatives
- Logit choice probabilities and analytic expectations

Consumer choice tree



Car owners' trading problem

$$V(i,a,\epsilon) = \max \left\{ \begin{array}{l} v(i,a,\kappa) + \epsilon(\kappa); \\ \max_{s\in\{1_s,0_s\}} \left[v(i,a,\emptyset,s) + \epsilon(\emptyset,s) \right]; \\ \max_{\substack{j\in\{1,\dots,J\}, \\ d\in\{0,1,\dots,\bar{a}-1\}, \\ s\in\{1_s,0_s\}}} \left[v(i,a,j,d,s) + \epsilon(j,d,s) \right] \end{array} \right\}$$

States Choices

- Existing car (i, a), traded car (j, d)
- When existing car (i, a) is replaced, there is additional scrappage choice $s \in \{0_s, 1_s\}$: to sell or to scrap the replaced car.
- Similar recursive maximization problems for consumers with no car and owner of car of terminal age $\bar{a}$

Choice specific value functions

$$
\begin{aligned}
v(i,a,\emptyset,1_s) &= u(\emptyset) + \mu \underline{P}_i + \beta EV(\emptyset) \\
v(i,a,\emptyset,0_s) &= u(\emptyset) + \mu[P_{ia} - T_s(P,i,a)] + \beta EV(\emptyset) \\
v(i,a,\kappa) &= u(i,a) + \beta\big(1-\alpha\big) EV(i,a+1) + \beta\alpha EV(i,\bar{a}) \\
v(i,a,j,d,1_s) &= u(j,d) - \mu[P_{jd} - \underline{P}_i + T_b(P,j,d)] + \\
&\quad + \beta\big(1-\alpha\big) EV(j,d+1) + \beta\alpha EV(j,\bar{a}) \\
v(i,a,j,d,0_s) &= u(j,d) - \mu[P_{jd} - P_{ia} + T_s(P,i,a) + T_b(P,j,d)] + \\
&\quad + \beta\big(1-\alpha\big) EV(j,d+1) + \beta\alpha EV(j,\bar{a})
\end{aligned}
$$

States Choices $\rightarrow$ Current period utility Future value

- Similar expressions for consumers with no car and owner of car of terminal age $\bar{a}$

Solving the consumers' problem

$$EV(i,a) = \sigma \log \left\{ \sum_{j,d,s} \exp \left[\frac{v(i,a,j,d,s)}{\sigma} \right] \right\}$$

- **Fixed point of Bellman operator in *EV* space**

$$EV(P) = \Gamma\big(EV(P), P\big)$$

- Conditional choice probabilities are then analitical, similar to

$$\Pi(j,d,s|i,a) = \frac{\exp\left[v(i,a,j,d,s)/\sigma\right]}{\sum_{j'} \exp\left[v(i,a,j',d',s')/\sigma\right]}.$$

  - *Note:* CCPs implicitly depend on car prices, P
  - The sell/scrap decision s is separable (see paper for details)
  - Fixed point solved using gradient-based Newton method with very precise starting values

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Stationary flow market equilibrium framework

Assumptions

1. Infinitely inelastic supply of new cars $\overline{P}_j$
2. Infinitely elastic demand for scrapped cars $\underline{P}_j$
3. $J(\bar{a}-1)$ endogenously determined used car prices P_{jd}

Definition: Ownership Distribution

$$q = \Big(\underbrace{(q_{11}, \ldots, q_{1\bar{a}})}_{\text{car } 1}, \ldots, \underbrace{(q_{J1}, \ldots, q_{J\bar{a}})}_{\text{car } J}, \underbrace{q_{\emptyset}}_{\text{no car}}\Big) \in \mathbb{R}^{J\bar{a}+1}$$

- q_{ia} is the fraction of consumers holding car i of age a
- By our timing assumption new cars purchased in any time period are accounted for as one-years-old cars in the next time period (so q_{j0} is undefined)

Equilibrium

Definition: Stationary Equilibrium

A pair $q^\star \in \mathbb{R}^{J\bar{a}+1}$ and $P^\star \in \mathbb{R}^{J(\bar{a}-1)}$ such that

1. Consumers maximize expected discounted utility,
2. Secondary market clears for all tradeable cars,
3. Ownership distribution is time-invariant.

The dynamics of the ownership distribution q are described by

- *Trade* transition probability matrix $\Omega(P)$ composed of conditional choice probabilities of trading decisions
- *Physical* transition probability matrix Q: ageing of cars + stochastic transitions to terminal age $\bar{a}$ (involuntary scrappage)

In the paper we also prove the *flow property* of this stationary equilibrium: all cars scrapped in each period are replenished by the exact amount of new cars bought in the same period

Trade transition probability matrix

$\Omega(P) =$ $\qquad$ $J\bar{a}+1 \times J\bar{a}+1$ matrix

$$
\begin{bmatrix}
\Delta_{11}(P)+\Lambda_1(P) & \Delta_{12}(P) & \dots & \Delta_{1J}(P) & \Delta_{1\o}(P) \\
\Delta_{21}(P) & \Delta_{22}(P)+\Lambda_2(P) & \dots & \Delta_{2J}(P) & \Delta_{2\o}(P) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\Delta_{J1}(P) & \Delta_{J2}(P) & \dots & \Delta_{JJ}(P)+\Lambda_J(P) & \Delta_{J\o}(P) \\
\Delta_{\o 1}(P) & \Delta_{\o 2}(P) & \dots & \Delta_{\o J}(P) & \Pi(\o|\o,P)
\end{bmatrix}
$$

Then $q \cdot \Omega(P)$ is distribution of cars after the trading phase

- $\Delta_{ij}(P)$ composed of choice probabilities of trading car i to car j
- $\Lambda_i(P)$ composed of keeping probabilities for car i
- $\Pi(\o|\o,P)$ is probability to remain in the no car state

Physical transition probability matrix

$Q =$ $J\bar{a}+1 \times J\bar{a}+1$ matrix

$$\begin{bmatrix} Q_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & Q_2 & 0 & \dots & 0 & 0 \\ 0 & 0 & Q_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \dots & Q_J & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix} \quad Q_j = \begin{bmatrix} 0 & 1-\alpha_j & \dots & 0 & \alpha_j \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1-\alpha_j & \alpha_j \\ 0 & 0 & \dots & 0 & 1 \\ 1-\alpha_j & 0 & \dots & 0 & \alpha_j \end{bmatrix}$$

- Aging of cars with probability $1 - \alpha_j$
- Total loss accidents with probability α_j
- Last row in each Q_j block is applies to the purchased new cars

$q \cdot \Omega(P)Q$ is ownership distribution in the next period

The stationary holdings distribution

$$\underbrace{q}_{t} \quad \rightarrow \quad \underbrace{q\Omega(P)}_{\text{after trading}} \quad \rightarrow \quad \underbrace{q\Omega(P)Q}_{t+1}$$

Condition for time invariance of the ownership distribution:

$$q = q\Omega(P)Q$$

Theorem (Uniqueness of stationary ownership distribution)

If scale of GEV shocks disctribution is positive then stationary ownership distribution is unique.

Proof.

Choice probabilities have full support $\Rightarrow$ transition matrix $\Omega(P)Q$ is irreducible and aperiodic $\Rightarrow$ uniqueness by the Fundamental theorem of Markov chains □

DNFXP algorithm (roadmap)

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Excess demand functions

- **Demand:** Fraction of consumers buying a given car (j, d):

$$D_{jd}(P, q) = \Pi(j, d|\emptyset, P)q_{\emptyset} + \sum_{i=1}^{J} \sum_{a=1}^{\bar{a}} \Pi(j, d|i, a, P)q_{ia}$$

- **Supply:** Fraction of owners that sell (not scrap) their car (j, d)

$$S_{jd}(P, q) = \big(1 - \Pi(\kappa|j, d, P)\big)\big(1 - \Pi(1_s|j, d, P)\big)q_{jd}$$

- **Market clearing condition** is the non-linear system of equations in ownership shares q and prices P

$$ED\big(P, q\big) \equiv D(P, q) - S(P, q) = 0$$

  - Given the stationarity condition $q = q(P)$
  - $J(\bar{a} - 1)$ equations with $J(\bar{a} - 1)$ unknowns

Existence of stationary equilibrium

Theorem (Equilibrium existence)

The stationary equilibrium for the automobile economy with the idiosyncratically heterogeneous consumers $(q^\star, P^\star)$ *exists, and in equilibrium it holds:*

$$
\begin{aligned}
q^\star &= q^\star \Omega(P^\star) Q, \\
0 &= ED(P^\star, q^\star).
\end{aligned}
$$

- Only existence: $q^\star$ is unique, but unclear about $P^\star$
- However, have not seen any signs of multiplicity in computations

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How to compute stationary flow equilibrium quickly?

Solving non-linear system of equations:

- **Gradient-based** solver! (in a series of lemmas show that all major objects in the model are smooth functions of prices, see paper)
- **Analytic derivatives**
- **Precise starting values** from the solution of the similar problem without transaction costs (linear system of equations, see appendix)

Newton method is therefore applied:

1. When solving the DP problem (Newton-Kantorovich)
2. When solving for equilibrium prices
3. When maximizing likelihood

- Chain rule of calculus used everywhere to build up gradients from already computed parts
- Run time in seconds for reasonable size problems on a laptop using unoptimized Matlab implementation

Adding persistent consumer heterogeneity

We extend the model to allow for sevel *types* of consumer heterogeneity:
• time-invariant • time-variant • combination of the two

- Existence theorems
- Computational algorithm is linear in the number of types
- Allows for sorting of consumers into the ages and types of cars
  - Rich hold newer better cars, poor hold older worse cars
  - Gains from trade and longer surviving cars
- The equilibrium conditions change only slightly

$$\text{Stationarity by type:} \quad \forall \tau \; q^\star_\tau = q^\star_\tau \Omega_\tau(P^\star) Q$$

$$\text{Market clearing in a sum:} \quad 0 = \sum_{\tau=1}^{N} f_\tau ED(P^\star, q^\star_\tau).$$

- Market clearing condition integrated over types

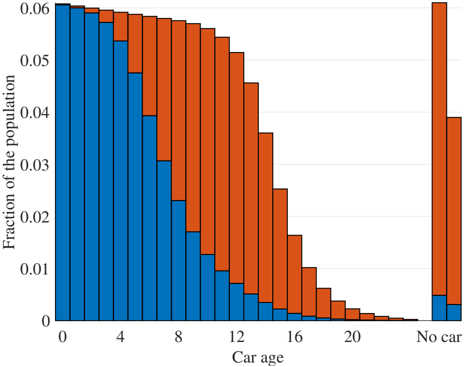
Gains from trade between rich and poor consumers

Rich mans Volvo

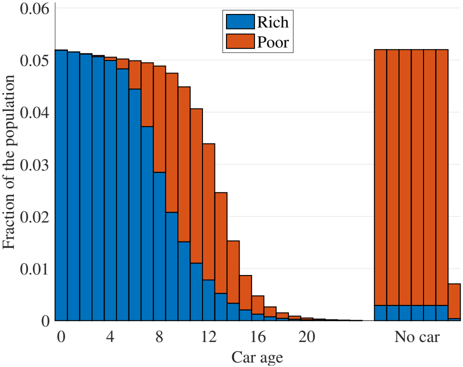
Poor mans Volvo

Illustrative example: ownership by two consumer types

Normal transactions costs



High transactions costs



- Sorting of consumers in each regime
- Heterogeneous effects of transaction costs

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Doubly Nested Fixed Point MLE estimator

- **Data:** counts $N_{x'x\tau}$ of transitions of household from state x (combining ownership and observable characteristics) to x' by the observed types τ
- Let θ denote the vector of structural parameters
- Transition probability $\Pi(x'|x,\tau,\theta)$ of the observed household state x composed of choice and transition probabilities at θ (see paper for details)
- Likelihood function

$$L(\theta) = \sum_{\tau} \sum_{x'} \sum_{x} N_{x'x\tau} \log \Pi(x'|x,\tau,\theta)$$

- Analytic gradient of the likelihood function again relies on using chain rule of calculus and already computed derivatives
- BHHH algorithm for approximation of Hessian

Part II: Modeling the Danish secondary market for automobiles

Simulating the effects of a hypothetical tax reform

Proposed Danish IRUC reform:

- lowers registration taxes, and
- raises usage taxes (road charging or gas tax).

Outcomes of interest:

- Equilibrium dynamics of car ownership and type choice:
  - new car sales and trade in secondary markets
  - fleet age and scrappage
  - value of the car stock
- Driving, fuel demand, and emissions
- Redistribution and welfare
- Need to capture these effects simultaneously

To implement the counterfactual simulation:

1. Estimate the model using Danish register data
2. Cut the registration tax rates for new vehicles by half
3. Increase the fuel tax rate such that revenue is unchanged
4. Compute economic/welfare/environmental implications

Utility specification with driving

Consider a utility function (indexes i and τ dropped)

$$u(a, x) = u_{\text{car}}(a) + u_{\text{drive}}(a, x) + \mu[\text{trade} + \text{transaction cost}]$$

$$\text{Onwership utility: } u_{\text{car}}(a) = \alpha_0 + \alpha_1 a + \alpha_2 a^2$$

$$\text{Utility from driving: } u_{\text{drive}}(a, x) = (\gamma_0 + \gamma_1 a)x - \mu p x + \frac{\phi}{2} x^2$$

- x is kilometers driven, p is cost per kilometer inclusive of tax
- parameters may be specific to car type i and consumer type τ
- See paper and online appendix for the estimated values of parameters

Assumption

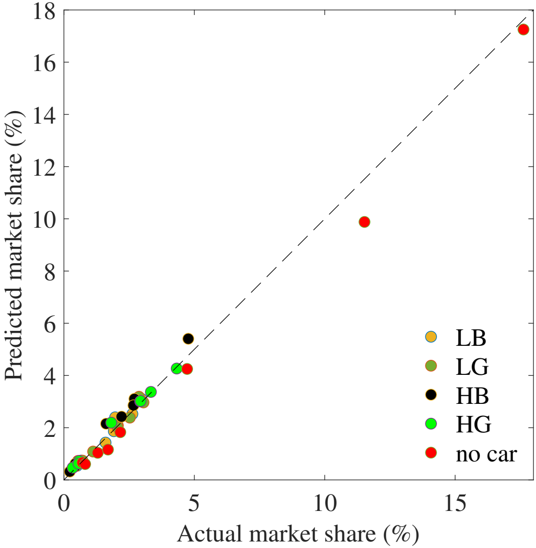
The probability of an accident and other physical deterioration in an automobile is independent of the amount of driving x.

$\Rightarrow$ driving is a static subproblem of the overall DP problem that can be solved independently

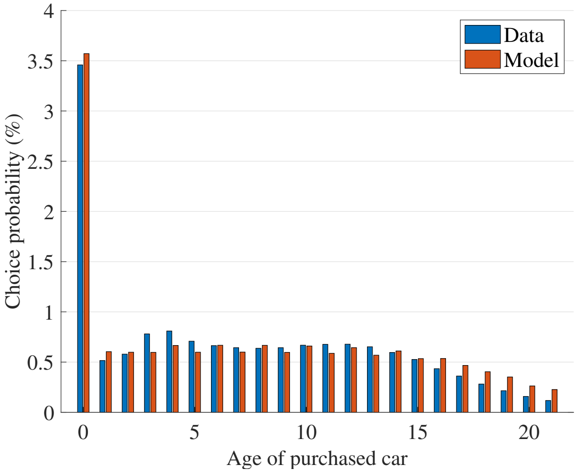
Model captures key features of Danish households

- Poor households are significantly more likely not to own a car than rich ones which are also willing to pay more for any type of car
- Couples are more likely to own cars and generally have higher willingness to pay for cars than singles
- High work distance households are relatively more likely to own cars and have higher willingness to pay for cars than those with low work distance
- All households preferred the heavy cars to the light ones and brown cars to green ones:
  heavy brown $\succ$ heavy green $\succ$ light brown $\succ$ light green
- Households with high work distance drive much more than those with low, and more so for the rich
- Model implies fuel price elasticities between -0.10 and -0.60 across households, similar to Gillingham and Munk-Nielsen (2015)

Model fit: Household-specific market shares

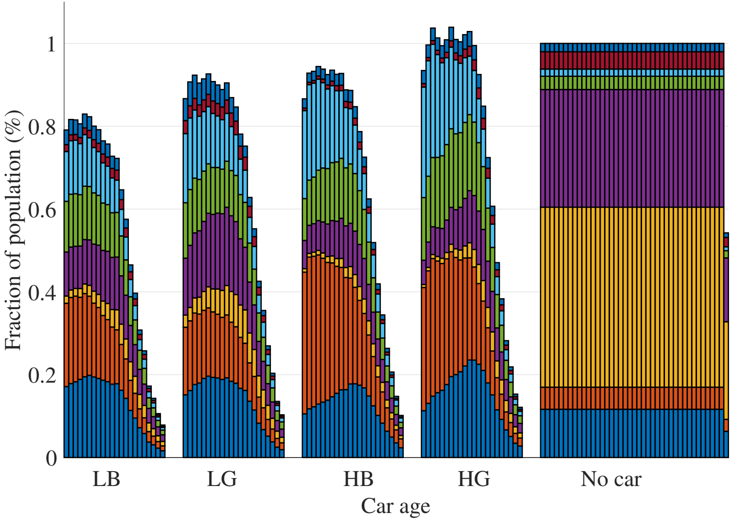


Model fit: Actual and predicted probability purchase

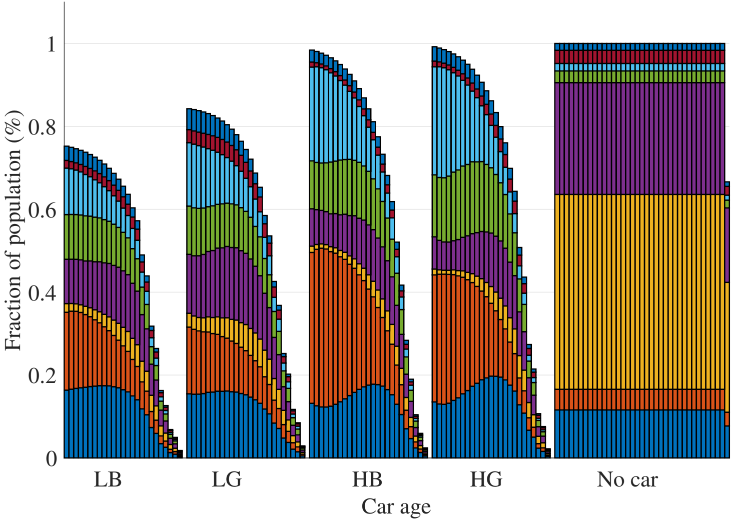


(a) Probability of purchase

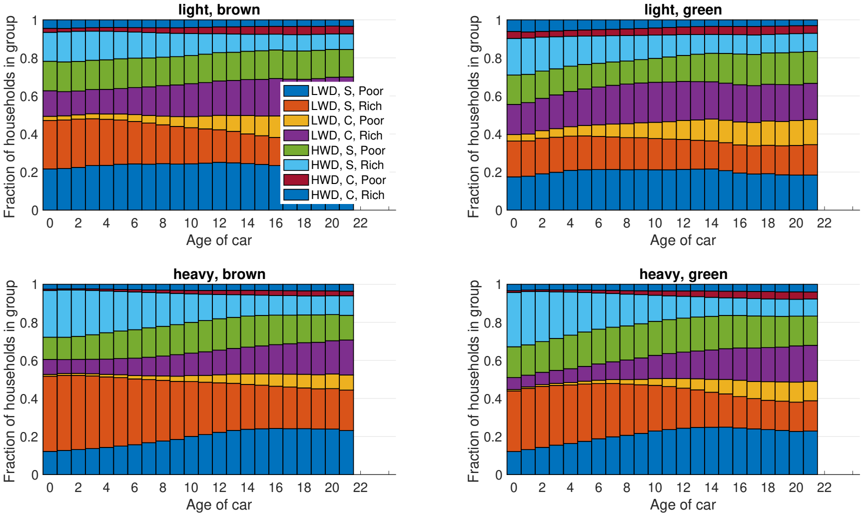
Model fit: Observed ownership distribution



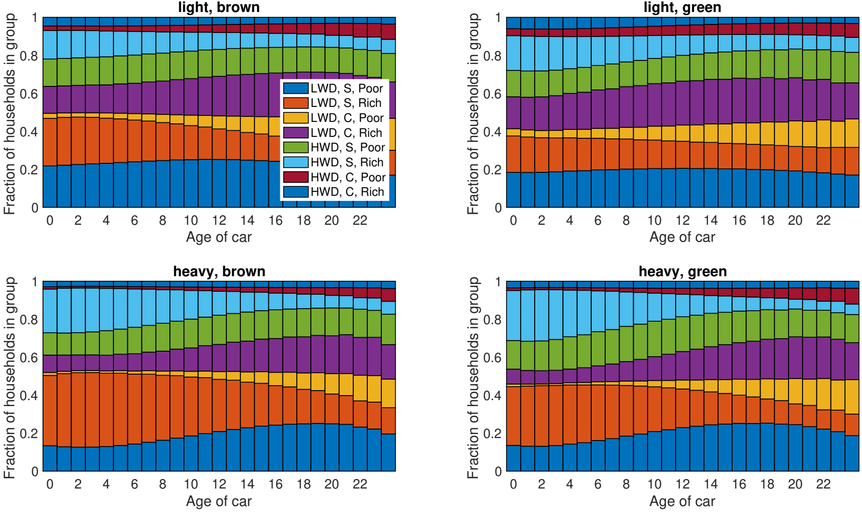
Model fit: Predicted ownership distribution



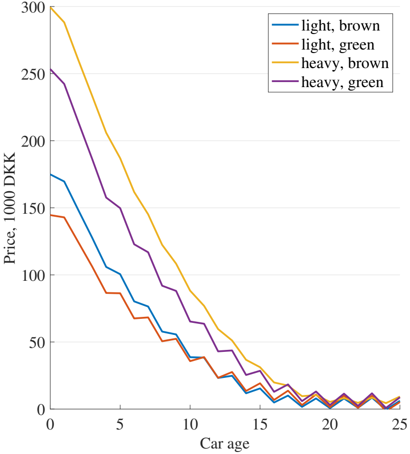
Model fit: Observed sorting



Model fit: Predicted sorting



Predicted equilibrium prices at secondary market



- Predicted prices similar to used car prices recommended by DAF.

Counterfactual simulation

Halving registration tax: Reduction in new car price between 25.6% (cheapest car), 28,6% (most expensive car)

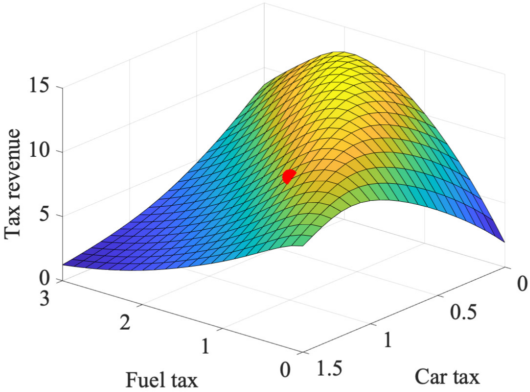
We consider the following four scenarios:

1. **Baseline:** Calibration under Danish tax rates from 2008.
2. **Naive, expected:** Non-equilibrium simulation:
   - Assume new and used car prices drops proportionally
   - Increase fuel taxes to keep total tax revenue neutral
     (Fuel price increase from 56% to 76% of the price at the pump)
3. **Naive, realized:** Equilibrium simulation:
   - Policy as above + market equilibrium imposed
   - Not revenue neutral in equilibrium
     (20% lower revenue than expected!)
4. **Sophisticated policy maker:**
   - Policy revenue-neutral in equilibrium
   - Fuel tax is lower, but leads to higher total tax revenue
     (compared to realized tax revenue for naive policymaker)

Policy Simulation Results

	Baseline	Naive, expected	Naive, realized	Sophisticated
Policy choice variables				
Registration tax (bottom rate)	1.050	0.525	0.525	0.525
Registration tax (top rate)	1.800	0.900	0.900	0.900
Fuel tax (share of pump price)	0.573	0.761	0.761	0.732
Prices				
Price, light, brown (1000 DKK)	174.902	129.532	129.532	129.532
Price, light, green (1000 DKK)	144.551	107.532	107.532	107.532
Price, heavy, brown (1000 DKK)	299.452	214.048	214.048	214.048
Price, heavy, green (1000 DKK)	253.397	182.796	182.796	182.796
Fuel price (DKK/l)	8.322	14.885	14.885	13.243
Outcomes				
Social surplus (1000 DKK)	9.382	11.281	8.439	10.203
Total tax revenue (1000 DKK)	9.391	9.391	7.452	9.391
Fuel tax revenue (1000 DKK)	4.282	5.184	4.983	6.224
Car tax revenue (1000 DKK)	5.110	4.207	2.468	3.167
Non-CO2 externalities (1000 DKK)	6.751	3.385	3.281	4.711
Externalities (1000 DKK)	7.374	3.702	3.586	5.157
Consumer surplus (1000 DKK)	7.364	5.592	4.573	5.969
CO2 (ton)	2.148	1.094	1.052	1.537
Driving (1000 km)	10.861	5.446	5.279	7.580
E(car age)	6.507	3.080	4.336	5.417
Pr(no car)	0.367	0.535	0.534	0.418

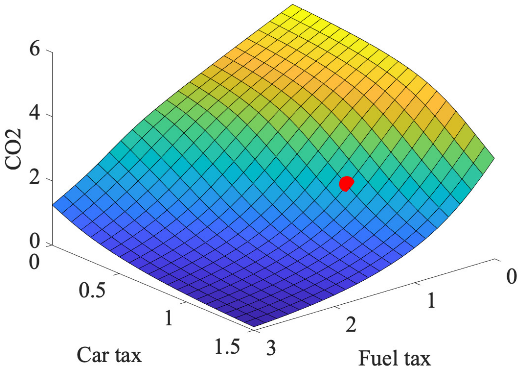
Laffer curves for new car registration tax and fuel tax



New car registration and the fuel tax relative to the baseline level of 1.

Tax revenue from new car sales tax and fuel tax.

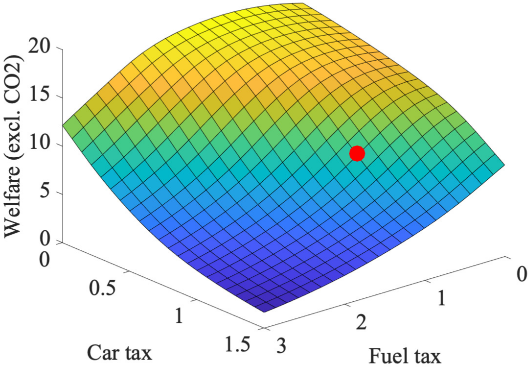
CO_2 emissions vs. new car registration and fuel taxes



New car registration and the fuel tax relative to the baseline level of 1.

Tax revenue from new car sales tax and fuel tax.

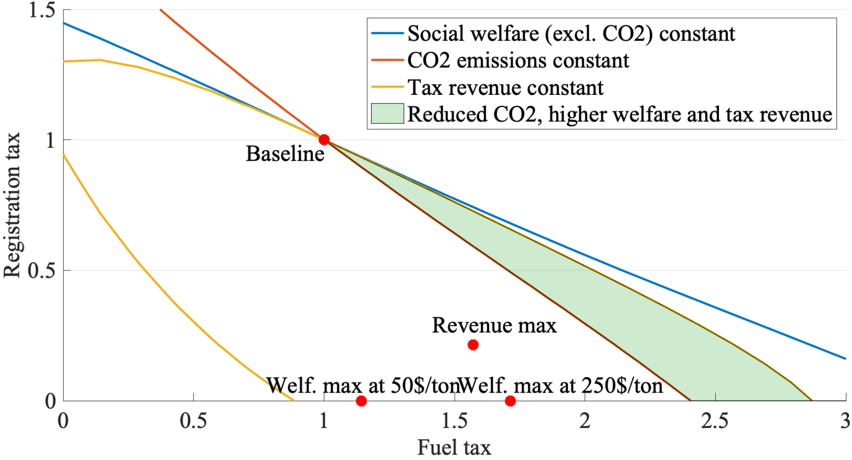
Social welfare (ex CO_2) vs. registration and fuel taxes



New car registration and the fuel tax relative to the baseline level of 1.

Tax revenue from new car sales tax and fuel tax.

Trade-off between CO_2 emissions and social welfare



Conclusion

- **Theory contribution:** characterize and prove existence of equilibrium in a tractable model of primary and secondary markets.
- **Applied contribution:** tractable model with
  - Transactions, scrappage, consumer/car heterogeneity,
  - Flexible utility: estimating 131 parameters with $39 \cdot 10^6$ observations in under 30 min on a laptop
- **Conclusion:** High Danish taxes above the Laffer curve's top point
  - "naive" model overestimates the strength of this effect,
  - possibly leading to detrimental policies for tax revenues and the environment
  - Opportunity to reduce CO_2, and increase tax revenues and social welfare